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Abstract—In the smart grid, which is a new generation grid integrated with bidirectional communication and advanced information technology, it is supposed that the number of energy suppliers will increase because many renewable resources will be connected to the grid. The increase in the number of suppliers that use renewable energy resources introduces new challenges in scheduling and dispatching controllable resources to control the load of the power grid in the system. By using the pricing method that maximizes social welfare, consumers/suppliers can determine the optimal power consumption/generation that maximizes own welfare. However, when a forecast error of renewable energy output occurs, the gap between the total power consumption and generation causes unless consumers decrease power consumption from the optimal consumption. Decreasing consumers’ power consumption might lower their utility. In this paper, we define impact on consumers’ utility and propose two demand response methods which aim to minimize the impact. The first method, called One-Time Demand Response, minimizes the impact on consumers’ utility of a certain time slot by using a Lagrange multiplier. In the second method, called Foresight Demand Response, all consumers can forecast the electricity market price of the next time slot by using linear regression, and consumers do not reduce power consumption when the incentive of a certain time slot is cheaper than the forecast electricity market price of the next time slot. From the performance evaluation, we confirm the impact on consumers’ utility in One-Time Demand Response and Foresight Demand Response. We also confirm that both two proposals can increase consumers’ welfare compared to not carrying out demand response.

Nomenclature

Indices and Numbers

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>Consumer Index.</td>
</tr>
<tr>
<td>N</td>
<td>Total Number of consumers.</td>
</tr>
<tr>
<td>j</td>
<td>Supplier Index.</td>
</tr>
<tr>
<td>M</td>
<td>Total Number of suppliers.</td>
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</tbody>
</table>

Variables

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>λ</td>
<td>Unit price at the market phase.</td>
</tr>
<tr>
<td>x_i</td>
<td>Power consumption of consumer i.</td>
</tr>
<tr>
<td>ω_i</td>
<td>Objective consumption of consumer i.</td>
</tr>
<tr>
<td>α_i</td>
<td>Parameter of utility function U_i.</td>
</tr>
<tr>
<td>̂x_i</td>
<td>Power reduction of consumer i determined in the demand response phase.</td>
</tr>
<tr>
<td>x_i^*</td>
<td>Power consumption of consumer i determined in the market phase.</td>
</tr>
</tbody>
</table>

Functions

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>U_i</td>
<td>Consumers’ utility function.</td>
</tr>
<tr>
<td>I</td>
<td>Consumers’ impact function.</td>
</tr>
<tr>
<td>C_j</td>
<td>Suppliers’ cost function for generating power.</td>
</tr>
<tr>
<td>W_i</td>
<td>Consumers’ welfare function.</td>
</tr>
<tr>
<td>W_j</td>
<td>Suppliers’ welfare function.</td>
</tr>
<tr>
<td>L</td>
<td>Lagrange function.</td>
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</table>

Constants

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>Step size in gradient method.</td>
<td></td>
</tr>
<tr>
<td>Terminate condition in gradient method.</td>
<td></td>
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<tr>
<td>Imbalance price.</td>
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</tbody>
</table>

One-Time Demand Response

\[ \hat{X} \]  Total amount of requested power reduction.
\[ \lambda \]  Lagrange multiplier, which means the incentive.
\[ p^f \]  Forecast market price.
\[ y^f \]  Forecast power generation.
\[ \beta_0, \beta_1 \]  Parameters of liner regression.
\[ \lambda^* \]  Market price determined in the market phase.

I. INTRODUCTION

Lately, many researchers focus on the smart grid, which provides integration of bidirectional communication and information technology. In the existing power grid, large-scale suppliers generate and distribute power in one direction from suppliers to consumers. Compared to this, the smart grid controls power demand and supply by using information and communication technology, and it will be able to increase the efficiency of energy usage in the smart grid. It is also supposed that generators or solar panels located in houses and buildings can connect to the smart grid as power suppliers, so there are many small suppliers in the smart grid. As a result of increasing suppliers, the relationship between the number of suppliers and consumers will change from 1 : N to M : N. From the market mechanism, the market price of energy might be the equilibrium price which balances power supply and demand. In addition to the market mechanism, the market price should be determined considering the welfare of both consumers and suppliers.

Samadi et.al propose the algorithm to determine the market price with maximizing social welfare [1], [2]. Social welfare is sum of the welfare of all consumers and suppliers. Samadi’s
proposal only supposes the system which has single supplier and multiple consumers, so Deng et al. propose the expansion of Samadi’s idea [3]. By using Deng’s algorithm, we can determine the market price, the amount of consumption, and the amount of generation, but the proposal does not consider suppliers that generate power by renewable energy resources. Considering renewable energy resources, it might not be able to generate the amount of power determined by Deng’s algorithm because the forecast power generation sometimes contains errors. If the real power generation is less than the forecast power generation, small suppliers need to buy power from large-scale suppliers and pay imbalance rate. Generally, imbalance rate is much higher than the market price.

On the other hand, there are some researches to cancel the forecast error by demand response [4], [5]. In these researches, consumers who reduce their power demand receive incentive according to the amount of their reduction. If the total incentive which suppliers pay to all consumers is lower than the imbalance rate, the welfare of suppliers increases by demand response. However, the impact on consumers’ utility must be considered since demand response will decrease power consumption of consumers.

In this paper, we propose two demand response considering the impact on consumers’ utility. Our proposals, called One-Time Demand Response and Foresight Demand Response, suppose that there are many suppliers.

The rest of this paper is organized as follows. In section II, we describe our supposed power market. We define the impact on consumers’ utility and explain our two proposals in section III. Section IV shows the results of simulation. We evaluate the change of consumers’ and suppliers’ welfare and the impact on the consumers’ utility. In the end of this paper, we present the conclusion in section V.

II. PROPOSED POWER MARKET

In this section, we illustrate the flow of power trading we propose. There are two phases in the trading: a market phase and a demand response phase.

First, in a market phase, the market price of energy is determined so as to maximize social welfare. To determine the price, we use the algorithm proposed by Deng et al. [3]. Then, consumers and suppliers decide how much they will consume or generate power based on the determined market price. We suppose that the maximum amount of power generated by renewable energy generators is same as the amount of forecast power generation at time $t$. The power consumption of consumer $i$ is determined by the following expression.

$$x_i(\lambda) = \arg \max U_i(x_i, \omega_i) - \lambda x_i$$  \hspace{1cm} (1)

$x_i(\lambda)$ means the consumption of consumer $i$ when the market price is $\lambda$. $U_i(x_i, \omega_i)$ is the utility function that expresses the satisfaction when consumer $i$ consumes power $x_i$. $\omega_i$ shows the objective consumption of consumer $i$. In our research, the utility function $U_i(x_i, \omega_i)$ is defined in the same way as [1], [2].

$$U_i(x_i, \omega_i) = \begin{cases} -\frac{\alpha_i}{2}(x_i - \omega_i)^2 & (0 \leq x_i \leq \omega_i) \\ 0 & (x_i \geq \omega_i) \end{cases}$$  \hspace{1cm} (2)

$\alpha_i$ is the parameter that each consumer individually has. Because $\lambda x_i$ is the payment of consumer $i$ when he or she consumes power $x_i$ at the unit price $\lambda$, the expression (1) means that satisfaction of consumer $i$ is the difference between the consumer’s satisfaction due to the power consumption and the dissatisfaction due to payments.

After a market phase, a demand response phase starts when forecast errors of renewable energy generation occur. In a demand response phase, the incentive needs to be determined that minimizes the total impact on all consumers’ utility at time $t$. In response to the determined incentive, each consumer decides how much to reduce its power consumption. We illustrate this method in the demand response phase in section III.

III. DEMAND RESPONSE MINIMIZING THE IMPACT ON THE CONSUMERS’ UTILITY

In this section, we first define and formulate the impact on consumers’ utility. Then, we propose two demand response methods; One-Time Demand Response and Foresight Demand Response. We summarize the features of two methods as follows.

- One-Time Demand Response: It is able to minimize the impact on consumers’ utility at time $t$.
- Foresight Demand Response: By forecasting the market price at time $t + 1$, consumer can enjoy further benefits.

Finally, we explain how both algorithms update the incentive values.

A. Impact on Consumers’ Utility

We define the impact on consumers’ utility as the difference in the utility function value before and after accepting reduction requests. We formulate the impact function $I(\hat{x}_i)$ as in (3).

$$I(\hat{x}_i) = U_i(x^*_i, \omega_i) - U_i((x^*_i - \hat{x}_i), \omega_i)$$  \hspace{1cm} (3)

$x^*_i$ is the power consumption of consumer $i$ determined in the market phase, and $\hat{x}_i$ is its power reduction in the demand response phase. Since consumers’ utility function is nondecreasing, the impact function $I(\hat{x}_i)$ will be greater than 0 while $\hat{x}_i > 0$. This means that consumers’ utility is always affected when the power consumption determined in the market phase is reduced as a result of accepting reduction requests.

Even if power reduction is the same amount, the impact on consumers’ utility is different depending on a consumer. In this paper, we propose demand response algorithms that minimize the total impact on consumers’ utility.
B. One-Time Demand Response

One-Time Demand Response aims to minimize the impact on consumers’ utility at time $t$. The objective function is the following:

$$
\text{minimize} \sum_{i \in N} I(\hat{x}_i) \quad (4)
$$

$$
s.t. \sum_{i \in N} \hat{x}_i = \hat{X} \quad (5)
$$

$\hat{X}$ means the amount of requested power reduction and is same as the amount of forecast error. $N$ is the set of consumers. The objective function (4) can be rewritten as follow:

$$
\text{maximize} \sum_{i \in N} -I(\hat{x}_i) \quad (6)
$$

$$
s.t. \sum_{i \in N} \hat{x}_i = \hat{X} \quad (7)
$$

The Lagrangian function is defined as in [6]:

$$
L = \sum_{i \in N} -I(\hat{x}_i) + \lambda \left( \sum_{i \in N} \hat{x}_i - \hat{X} \right)
$$

$$
= \sum_{i \in N} \left( U_i((x^*_i - \hat{x}_i), \omega_i) - U_i(x^*_i, \omega_i) \right) + \lambda \left( \sum_{i \in N} \hat{x}_i - \hat{X} \right)
$$

$$
= \sum_{i \in N} \left\{ U_i((x^*_i - \hat{x}_i), \omega_i) + \lambda \hat{x}_i - U_i(x^*_i, \omega_i) \right\} - \lambda \hat{X} \quad (8)
$$

$\lambda$ is a Lagrange multiplier and means the incentive. From (8), we are able to say that each consumer can calculate the amount of power reduction according to the incentive presented by suppliers. The following expression for consumers is used to calculate the power reduction:

$$
\hat{x}_i = \arg \max U_i((x^*_i - \hat{x}_i), \omega) + \lambda \hat{x}_i - U_i(x^*_i, \omega) \quad (9)
$$

Until the suppliers stop updating the incentive, consumers calculate the power reduction by using (9) and send the results to the suppliers.

C. Foresight Demand Response

Consumers need to buy back power at time $t + 1$ by the reduced amount at time $t$. In One-Time Demand Response, consumers do not forecast the market price at time $t + 1$, so they might lose money when the incentive at time $t$ is lower than the market price at time $t + 1$. Consumers have a strategy to solve this problem in Foresight Demand Response. The difference between two demand response algorithms is that consumers will accept a reduction request from suppliers at time $t$ only after the incentive becomes higher than the forecast market price at time $t + 1$. In Foresight Demand Response, the objective function and the way to determine the power reduction of consumer $i$ are same as those in One-Time Demand Response.

In this subsection, we explain the method to forecast the market price at time $t + 1$. Figure 1 shows the relationship between the forecast power generation and the market price. The forecast power generation is predicted by using the data taken from an actual 15kW-class solar power plant in October 2016, and simulating only a market phase with the forecast power generation. We do not explain the method to forecast the amount of power generation of a solar power plant in this paper. Each blue dot in Figure 1 shows the daily market price at 14:00 according to the amount of the forecast power generation. The red line in Figure 1 means the forecast line of the market price. From Figure 1, we confirm that the market price becomes cheaper as the amount of the forecast power generation becomes larger. We employ linear regression to forecast the market price. The line of linear regression is expressed as follows:

$$
p^f = \beta_0 \times y^f + \beta_1 \quad (10)
$$

In (10), $p^f$ and $y^f$ mean the forecast market price and the forecast power generation, respectively. $\beta_0$ and $\beta_1$ are the parameters determined by machine learning. In Foresight Demand Response, consumers never accept power reduction requests unless the proposed incentive becomes larger than $p^f$. Compared to the incentive in One-Time Demand Response, the incentive in Foresight Demand Response might rise, but suppliers send power reduction request to consumers while the incentive is under the imbalance price.

Since the amount of power reduction is determined by the incentive, the total power reduction of all consumers can be over the amount of the requested power reduction when the lower limit of the incentive accepted by consumers is the forecast market price. Because the presented incentive in One-Time Demand Response is smaller than that of Foresight Demand Response, the power reduction of each consumer is also smaller. Thus, many consumers can participate in One-Time Demand Response.

The number of consumers who participate in Foresight Demand Response is less than that of One-Time Demand Response. It is because only consumers who accept much power reduction participate in Foresight Demand Response. In other words, we can decrease the number of consumers whose utility will be affected.
D. The Incentive Update

At the end of this section, we explain the update method for the incentive. Suppliers update the incentive by using gradient method[6]. To determine the suitable incentive that balances the total reduction with the requested reduction, the following formula is used:

\[ \hat{\lambda}^{k+1} = \left[ \hat{\lambda}^k + \hat{\gamma} \left( \hat{X} - \hat{\lambda}^k \sum_{i \in N} \hat{x}_i \right) \right]^+ \]  (11)

\( k \) and \( \hat{\gamma} \) mean the count of iterations and the step size in gradient method, respectively. We show the following algorithm to decide the incentive.

\textbf{Algorithm 1} Executed by suppliers in the demand response phase.

\begin{algorithm}
\begin{algorithmic}
\Require \( \hat{X} \) is already known.
\State \( k \leftarrow 0 \)
\State \( \hat{\lambda}^0 \leftarrow 0 \)
\While {\( |\hat{\lambda}^{k+1} - \hat{\lambda}^k| \geq \hat{\varepsilon} \)}
\State Compute the new value of \( \hat{\lambda}^k \) using (11).
\State Send \( \hat{\lambda}^k \) to all consumers.
\State Receive \( \hat{x}_i^k \) from each consumer \( i \in N \).
\State Update the total reduction \( \hat{\lambda}^k \sum_{i \in N} \hat{x}_i^k \)
\State Increment \( k \)
\EndWhile
\end{algorithmic}
\end{algorithm}

First, the incentive \( \hat{\lambda}^0 \) is initialized to 0, and suppliers present the initial incentive \( \hat{\lambda}^0 \) to consumers. Each consumers calculates its desired power demand reduction \( \hat{x}_i^k \) using (9) according to the presented incentive \( \hat{\lambda}^k \). Then, each consumer sends own power demand reduction \( \hat{x}_i^k \) as a reply to suppliers. After receiving power reduction from all consumers, suppliers update the incentive by (11), and present the updated incentive \( \hat{\lambda}^{k+1} \) to consumers again. Suppliers iterate updating incentive until the difference between the \( k \)th incentive \( \hat{\lambda}^k \) and the updated incentive \( \hat{\lambda}^{k+1} \) becomes smaller than the termination \( \hat{\varepsilon} \).

IV. PERFORMANCE EVALUATION

In this section, we show performance evaluation about our proposal. We simulate welfare of consumers and suppliers and impact on consumers’ utility.

Table 1 shows our simulation parameters. Figure 2 shows hourly power demand of consumers. In our simulation, power demand of consumers in each time slots follows a normal distribution whose average is the value in the corresponding time slot in Figure 2 and the standard deviation is 100.

A. Consumers’ welfare

We use the utility function \( U_i(x_i, \omega_i) \) of consumer \( i \) as shown in section II. The consumer’s welfare function \( W_i \) is defined as:

\[ W_i = U_i((x_i^* - \hat{x}_i), \omega_i) - \lambda^* (x_i^* - \hat{x}_i) + \hat{\lambda}\hat{x}_i \]  (12)

\( \lambda^* \) and \( \hat{\lambda} \) are the market price determined in the market phase and the incentive in the demand response phase, respectively.

Similarly, \( x_i^* \) and \( \hat{x}_i \) mean consumer \( i \)’s power demand determined in the market phase and consumer \( i \)’s power reduction determined in the demand response phase, respectively.

Figure 3 shows how the proposed demand response algorithms improve consumers’ welfare. For each time slot, bars on the left-hand side show impact on consumers’ welfare in One-Time Demand Response, and those on the right-hand
side show that of Foresight Demand Response. Each bar is divided into different colors to show the difference among the consumers. For example, in the 8th time slot (13:00 - 14:00), three consumers participated in Foresight Demand Response. Since impact on consumers’ welfare is calculated as the difference in the utility function value before and after participating in demand response, it can be confirmed that consumers’ welfare increases in both of the two proposals compared to not participating in demand response. In addition, Foresight Demand Response is able to increase consumers’ welfare three times as much as One-Time Demand Response. This is because the incentive in Foresight Demand Response is higher than that of One-Time Demand Response and the third term of (12) becomes bigger.

Furthermore, Figure 3 shows that the number of participating consumers in Foresight Demand Response is less than that of One-Time Demand Response. Consumers accept the power demand reduction only when the incentive is over the forecast market price of the next time. When suppliers present the incentive as same price as the forecast market price, the total amount of the power demand reduction is much higher than the amount of power reduction requested by the suppliers. Then, consumers who accept much power reduction will participate in the reduction of consumption preferentially. Therefore, the number of participating consumers in Foresight Demand Response is less than that in One-Time Demand Response.

B. Suppliers’ welfare

We use welfare function \( W_j \) of supplier \( j \) as followings:

\[
W_j = \lambda^*(y^m) - C_j(y^m) - \hat{\lambda} \sum_{i \in N} \hat{x}_i - \hat{\lambda}(\hat{X} - \sum_{i \in N} \hat{x}_i) \quad (13)
\]

\( \hat{\lambda} \) is the imbalance price shown in table I, and the function \( C_j \) means the cost for generating power. \( y^m \) shows the amount of the actual power generation. The first term of (13) means income by selling electricity, the third term is the payment to all consumers, and the fourth term means the imbalance fee.

Figure 4 shows how the proposed demand response algorithms impact suppliers’ welfare. For each time slot, bars on the left-hand side show impact on suppliers’ welfare in One-Time Demand Response, and those on the right-hand side show that of Foresight Demand Response. Since consumers may not accept the power reduction requests from suppliers in Foresight Demand Response, the suppliers’ welfare in One-Time Demand Response is larger than that in Foresight Demand Response. The main factor of this result is the third of (13). The incentive in Foresight Demand Response is higher than that in One-Time Demand Response because consumers have the strategy for increasing their welfare. Therefore, the third term of (13) becomes larger in Foresight Demand Response.

C. Impact on Consumers’ Utility

Figure 5 shows how the proposed demand response algorithms impact consumers’ utility. The relative position and color of each bar in a time slot has the same meaning as in Figure 3. Impact on consumers’ utility is calculated by (3), and the quality of consumers’ lives is lowered as the value of impact becomes larger. From Figure 3 and 5, we confirm that the consumers who get much satisfaction with a large incentive lower their utility notably. Because the utility function \( U_i(x_i, \omega_i) \) is nondecreasing and consumers decrease their power consumption by demand response, the quality of consumers lives must be lowered when consumers agree to participate in demand response. In conclusion, One-Time Demand Response is the way to minimize the impact on consumers’ utility, and Foresight Demand Response increases the impact on consumers’ utility by 25% because consumers have the trading strategy.

V. CONCLUSION

In the smart grid, a lot of power plants using renewable energy will be introduced, and the relationship between the number of suppliers and consumers will change from 1 : \( N \) to
Suppliers generating power by renewable energy have to forecast their amount of power generation, but a forecast error might occur. Demand response is a way to cancel out the forecast error. In demand response, consumers decrease their power consumption and lower quality of their lives instead of receiving the incentive. In this paper, we define the impact on consumers’ utility, and propose two demand response algorithms minimizing the impact. One-Time Demand Response can minimize the impact on consumers’ utility at time \( t \). In Foresight Demand Response, consumers can receive the higher incentive because they forecast the market price at time \( t + 1 \). Through performance evaluation, we confirm that both proposals can increase consumers’ welfare and Foresight Demand Response is able to increase consumers’ welfare three times as much as One-Time Demand Response. However, there remains a problem that suppliers’ welfare decreases in Foresight Demand Response because of consumers’ trading strategy. It is also problem that Foresight Demand Response impacts a few consumers’ utility considerably.

REFERENCES


